Science Education for New Civic Engagements and Responsibilities

Mathematical and Statistical Reasoning in Compelling Contexts: Quantitative Approaches for Building and Interrogating Personal, Disciplinary, Interdisciplinary and Worldviews

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A SENCER BACKGROUNDER

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In 1992, Professor Ferguson received the State University of New York Chancellor's Award for Excellence in Teaching. He is a New York State and national leader in programs to enhance the participation of underrepresented minority students in science, technology, engineering and mathematics programs. He is director of both the NSF-supported SUNY Louis Stokes Alliance for Minority Participation and the NSF-supported SUNY Alliance for Graduate Education and the Professoriate. In 1997, he received the U.S. Presidential Award for Excellence in Science, Mathematics and Engineering Mentoring. Professor Ferguson served as a member of the executive committee of the NSF-supported Recognition Award for the Integration of Research and Education (RAIRE) at Stony Brook. From 1998 to 2002, Professor Ferguson directed Stony Brook's Center for Excellence in Learning and Teaching (CELT). The New York Academy of Sciences named Dr. Ferguson as the Academy's 2004 recipient of its Archie Lacey Award, which is presented nationally to an individual who has made extraordinary contributions to the participation of underrepresented minority students in STEM fields.

I. Introduction

Many students with interests outside of "quantitative fields" perceive much of school and college mathematics as dull, difficult, scary and totally irrelevant. For these students, mathematics is a "necessary evil," or a dreaded game, rather than a subject to be enjoyed and integrated into their personal and professional lives. What, then, are the challenges of bringing mathematical and statistical reasoning into richer contexts so that students with multiple interests, strengths, purposes and worldviews might see the beauty and understand the uses and abuses of quantitative approaches? Such a reformed view of mathematics and its place in our culture not only benefits individual students. I believe that it is also essential for our economy and democracy.

In this paper, I attempt to describe an emerging vision for mathematical and statistical reasoning. That vision situates quantitative reasoning within broader contexts with the aim of bringing about a dramatic increase in the number of students who see such reasoning as an integral part of their studies, future careers, evolving artistic sense, and participation in a democracy. "Mathematics and statistics courses" is not the primary focus of this paper — though such courses are likely to be one of the vehicles through which this emerging vision for quantitative thinking may be addressed. Drawing upon the science of learning and emerging practices, I present a vision of mathematical and statistical reasoning by characterizing key elements of a reformed learning environment. I illustrate some of the key elements through examples from a course that I teach with the theme of "Introduction to Modeling and Decision Making" (IMDM).

In IMDM, I seek to situate mathematics and statistics within a broader context. The unifying theme of the course is "quantitative modeling." In this context, a "model" is considered to be a tentative conceptualization of reality that helps us to better understand or predict phenomena. In the course, students are taught to seek answers to a variety of questions as they attempt to build, interpret or assess quantitative models.

- **What is the purpose of the model?** (What kinds of questions will the model help you answer?)
- **What is the quality of basic data?** (What role will this data play in building the model and assessing the validity of the model?)
- **What assumptions are made?** (Are the simplifying assumptions reasonable?)
- **What factors are incorporated into the model?** (What factors are ignored? Why were these factors ignored?)
- **What mathematical relationships are used?** (What weights are given to the different factors? How are the factors related? What is the basis for using a certain set of equations?, etc.)
- **How would you test the quality of the model?** (How "good" is "good enough?")
- **What are competing models?** (How will you decide among all models that purport to address similar issues?)

In this paper, I explore key elements of a reformed environment for engaging students in mathematical and statistical reasoning. In the next section, we explore the role of motivation in learning and consider various motivations for students' learning of mathematics and statistics. Section III, entitled "Rigor Versus Rigor Mortis," challenges assumptions about what constitutes rigor. Section IV looks at key features of a powerful learning environment. Section V situates our view of "compelling contexts" within the framework of the science of learning. In Section VI, we discuss design challenges, informal efforts, and research results related to three "compelling contexts:" mathematics as art, science-technology-society approaches, and solving non-routine problems. In Section VII, we conclude by exploring the challenges of moving from the current status of the learning of mathematics and statistics to a more engaged populace. Throughout this paper, I will draw heavily upon insights from the science of learning as presented by Etkina and Mestre in their 2004 SENCER backgrounder on "Implications of Learning Research for Teaching Science to Non-Science Majors" (Etkina and Mestre, 2004). I also draw upon related work in this area (Mestre, 2002, 2005; Mestre and Cocking, 2000).

II. Motivation for Learning

Rather than make mathematics and statistics integral parts of their college work, many students see courses in these areas as dreaded requirements. They often view mathematics and statistics as a collection of definitions and rules that are to be memorized and applied to toy problems. There are several negative consequences of this rule-focused approach to the learning of these subjects: (1) definitions and rules are easily forgotten; (2) students cannot readily connect concepts and approaches; (3) students are not able to apply knowledge in appropriate contexts; and (4) students come to see mathematics and statistics as a dreaded game that has little or no relevance to their personal and professional development. As Etkina and Mestre summarize, students often see little practical benefit of learning "school science" other than to get a good grade (Etkina and Mestre, 2004). Etkina and Mestre go on to point out that research demonstrates that the experience of autonomy and ownership of ideas is strongly associated with motivational gains (Stipek, 2002).

In the "Introduction to Modeling and Decision Making" (IMDM) course that I teach, special effort is made to provide multiple linkages to students' areas of interest. For example, the concept of modeling has applications in every discipline and every profession. There are many ways of "knowing" the world. Quantitative modeling is viewed as a powerful way of exploring the world. The process of modeling can become an integral part of students' thinking.

Students should also come to realize that "more quantitative" is not always synonymous with "better." There are many examples where inappropriate or inadequate formulations lead to models that are of little value in helping us to understand or predict phenomena. I have used the following as a striking example in the introduction to discussion of inadequate modeling:

> In the mid-1960's, a Dutch hen farmer ordered large numbers of chickens from a Midwestern poultry farm. He chartered a 707 jet airplane (normally holding 180 passengers and crew) to bring the chickens to the Netherlands. The Midwestern farm and the airline wanted to transport the maximum number of chickens possible on a flight. The chickens averaged two pounds in weight, while human passengers average 170 pounds. Consequently, they decided to carry 170/2 x 180 or 15,300 chickens. (They estimated that the weight of the cages for the chickens would approximately match the typical weight of luggage for human passengers. There would be no passengers on the trip ─ just pilot and co-pilot.) Unfortunately, by the time that the 707 was in the air for 30 minutes, the pilot had to land the plane.

What was wrong with the model? Very few students (about one or two out of 100) are able to see that the model does not take into consideration the difference in basal metabolic rates between chickens and humans. Hence, they are not able to arrive at the conclusion that fog became a major problem inside the aircraft. This example provokes one to begin to think about the role of modeling in his/her own discipline. The author uses examples of models from a variety of sources.

I want students to understand that models are "artifacts" that may or may not be useful in understanding or predicting phenomena. During the first two or three weeks of IMDM, I present students with a variety of models in restricted contexts that will allow them to focus on the elements of the models as opposed to more general text. Models are drawn from such areas as population estimation (e.g., Laplace's method of estimating the population of France from birth records), medical testing, vaccines and risk, credit ratings, and even course grading as a modeling activity. A number of sources can be used for samples of "small" models (de Chadarenian and Hopwood, 2004; COMAP, 2003; Franchi, 2005; Fusari and Kenschaft, 2003; Giodana, Weir, and Fox, 2003; Greenfield, 2004; Harte, 1988, 2001; Judson, 1980; Moore, 2001; Newwirth and Arganbright, 2004; Shak, 2003; Starfield et al., 1990).

Generally, students will not encounter isolated models. In their studies, careers and other aspects of everyday life, they will experience models that are embedded within broader contexts. Here, they must be able to identify where "modeling" appears, and they must be able to comprehend and raise critical questions regarding the models. I use a variety of resources to engage students in studying modeling in rich domains. I have used the SENCER backgounders on HIV/AIDS (Keeling, 2003; Meyer, 2003), Hunger, Science and Public Policy (Hopkins, 2003), and Tuberculosis (Fluck, 2004). In addition, we have looked at modeling in the Consumer Price Index, estimates of credit card debt, the economics of resources, and a host of other issues: life expectancy across the world, infant mortality rates in various countries, and consumer finance, as well as the use of modeling in experiments and hypothesis testing.

Decision-making is an excellent theme for generating discussion on the appropriateness or inappropriateness of quantitative modeling in addressing personal, community and other issues. Decision-making is an interdisciplinary subject that includes mathematics and statistics, psychology, and sociology. I draw upon a variety of resources: Baron, 2000; Copp, 1989; Day, Reibstein and Gunther, 1997; Hock, Kunreuther and Gunther, 2001; Kahneman, Slovic and Tversky, 1982; McKenna, 1991; Nutt, 2002; Plous, 1993; Smith, 1990; Tavel, 1989; Truxal, 1989, 1991; Truxal and Visich, 1992.

Learning needs to be meaningful to the learner. Such meaning can be derived from situating knowledge within personal, disciplinary, multidisciplinary and worldviews. Both pure and applied mathematics can have meaning for students in this broader context. Although the motivations for doing pure and applied mathematics may differ, both areas can be situated in broader contexts that help students to make connections with their interests. Fundamentally, however, no matter how much the teacher "loves" the subject, one must be aware of the importance of the learner's motivation for learning in determining learning outcomes (Stipek, 2002).

Mathematics and statistics need to be situated in "compelling contexts" that facilitate students' frequent and deep engagement with the subjects. Such contexts will give serious attention to cognitive, affective, social and cultural dimensions of learning. Through such contexts, the concepts and approaches of the subject will be ever present, though sometimes not so transparent, in the students' evolving understanding of core ideas and their efforts to connect these ideas to new experiences. From elementary school through graduate school (and beyond), we need to include in our learning goals these rich interconnections of knowledge and enduring influence.

Let's take the case of calculus. What would it mean to situate calculus in a "compelling context?" Perhaps the notion of "rate of change" could be an important component of a "compelling context" for the learning and teaching of calculus. For the student taking calculus, there is a wealth of phenomena in related subjects (chemistry, biology, physics, economics, business, psychology, sociology, etc.) not to mention the daily news that can be drawn upon to illustrate the meaning of "rate of change" in multiple contexts. The author acknowledges that "rate of change" is already an important idea in calculus; however, the point here is that this notion is not adequately exploited so that students of diverse interests and backgrounds can explore it in personally meaningful contexts. In most calculus courses, "rate of change" is treated without context—as a "fact" you could say—or in a very narrow context, rather than within a host of potentially compelling (engaged, interconnected) contexts. The calculus reform movement addressed many related issues (Kaput, 1997; Knisley, 1997; Roberts, 1996; Solow, 1994; Tucker, 1995). "Compelling context" may be different for different types or levels of mathematics or statistics courses. For example, in a graduate mathematics course, context might relate to issues of connectivity (e.g., to other areas of mathematics). Even here, however, the issues of enduring influence and the needs and assets of community of learners are extremely important.

III. Rigor Versus Rigor Mortis

In the context of mathematics and statistics courses and papers, "rigor" often refers to the care that is taken in developing the subject ─ especially as it relates to system building, including presentation of definitions and theorems. A hierarchical presentation is often equated with a rigorous development. The assumption is that a student who has gaps in his/her knowledge may find it extremely difficult, or even impossible, to fill in the "missing pieces" at some later time. In addition to the hierarchical ordering of learning, such rigorous presentations often make use of lots of symbolic notations and rely heavily on much mathematical foundation. This method for developing knowledge of the subject can be invigorating for those students who have ample background and whose interests are in the kind of system building (pure or applied) that is enabled by such formalism. However, many students find it extremely difficult and unsatisfying to engage with this more formal view of mathematics. We can either live with the current rates of failure or we can explore more ways to engage a broader range of students with the beauty and uses of mathematics ─ even with those dimensions of mathematics that require sustained and rigorous development in the sense outlined above. To do the latter, we must be open to other notions of rigor that situate mathematical and statistical reasoning in broader contexts that allow more students to engage from positions of strength. From such broader engagement, we are likely to build mathematical competence and grow the number of students who are willing and able to engage mathematics at a more formal level.

Rigor has a special meaning in the context of the course on "Introduction to Modeling and Decision Making." The course has a modest number of mathematical concepts that are treated in a constructivist approach so that students develop a deep understanding of this core. Students with varying subject backgrounds, mathematical achievement levels, and career interests are able to engage with the core content. From the core content, students are able to move along self-initiated paths that allow them to (a) deepen their understanding of the core content, (b) further their learning of additional mathematics, and (c) integrate what they have learned into applied contexts that connect with their personal, disciplinary, and career interests.

In IMDM, the core mathematical and statistical content is situated in applied contexts. The six major subject content areas, together with the learning/teaching approach (in italics), are given below.

1. Modeling

Students study models of increasing complexity and of increasing embeddedness within complex arguments, including policy studies.

2. Principles and Applications of Probability

Students work with concrete examples to develop a personal understanding of the principles of probability. A given problem is often approached from several directions. For example, both joint-frequency tables and Bayes' theorem are used to solve certain problems involving conditional probability. Furthermore, principles of probability are applied in multiple "real-world" contexts, including quality control in industry, medical testing, and accident and injury cases.

3. Descriptive and Inferential Statistics

First, students learn to generate and explore properties of data by working with "real" data sets. Students learn properties of probability and come to see that the use of these properties may lead to counterintuitive results. The course draws on many resources on numeracy (Alonso and Starr, 1986; Bernstein, 1997, Bogue, 1990; Cohen, 1982/1999; Crosby, 1997; Dehaene, 1997; Desrosiéres, 1998; Helme and Marr, 1991; Hunon, 1998; Madison and Steen, 2003; Mack and Weisberg, 1992; Paulos, 1988; Steen, 1990/1991, 1997, 1999, 2001, 2004) *and statistics* (Bradstrect, 1996; Cassidy, 1969, 1984; Garfield, 1995; Gnanadesikan, Schaffer, and Watkins, 1997; Haskins and Jeffrey, 1990; Landmehr, Swift, and Watkins, 1987; Lindgren and Heather, 2001; Neuman, Obremski and Scheaffer, 1987; Porter, 1986; Scheaffer, Gnandesikan, Watkins, and Witmer, 1996; Schwartz, 1992; Tufte, 1990). *Students show a particular interest in models that are drawn from the environmental health field* (Berg, Hines, and Allen, 2002; Center for disease Control, 2001, 2002; Fine, 1996; Fisher, 2001; Guyer, 2001; Hill, 1965; Kamrin, 2003; Late, 2002; Moeller, 2005; National Research Council, 1996; Rhomberg, 1996).

4. The Mathematics of the Digital Revolution

For All Practical Purposes (COMAP, 2003) *provides excellent material in this area. I supplement the material with a bit on radio frequency identification (RFID). I ask students questions regarding the advantages or disadvantages of RFID as compared to bar codes.*

5. Finance and Resource Models

In addition to studying material in For All Practical Purposes (COMAP, 2003), *students study credit rating models and energy models.*

6. Quantitative Models in Decision Making

Each student selects a decision problem (personal, community, work, etc.) and analyzes the problem in order to make a recommendation regarding the "best" course of action. In implementing the project, students confront issues of data (quality, representation, summary, etc.) and modeling.

The treatment of the core content differs in three major ways from standard treatments: (1) students must treat the learning of math as they treat other complex domains where they must use a variety of resources ─ online math tutorials to fill in gaps in their backgrounds, other students as resources, etc.; (2) students must read current papers and engage in discussion on the uses and abuses of data and quantitative models in these papers; and (3) students must use their knowledge from the course to define, implement and describe their own projects that demonstrate the role of modeling in addressing personal, work or "community" decision problems. In the context of IMDM, rigor is achieved through making sense of core ideas and situating those ideas within student-initiated and student-engaged applied contexts.

The previous discussion is not at all meant to suggest that applied contexts are the only legitimate arenas to situate mathematical and statistical thinking for students with interests outside of "quantitative disciplines." The overarching argument is that context is important in both applied or pure mathematics contexts. Indeed, there is lots of evidence that mathematics has been taught and learned in ways that make it difficult for students to make sense of what they have learned. Such research points to the importance of viewing mathematics as a constructive activity. In this view, knowledge is formed from active interplay between learners' existing knowledge and new concepts. Such interaction is both cognitive and social (Bauersfeld, 1979); Brown, Collins and Duguid, 1989; Collins, Brown and Newman, 1989; Lave, Smith and Butler, 1989; Greeno, 1989; Resnick, 1989; Schoenfeld, 1989a; Schoenfeld, 1992). The approach outlined in this paper is the author's response to emerging understanding about how students come to understand mathematics and solve problems.

IV. The Learning Environment

In this section, we explore key features that I have tried to build into the total learning environment in the teaching of IMDM. These features are modified from a prior paper (Ferguson, 1993a). After describing a feature, I will share (in italics) an insight about the class as a whole or individual students related to that feature.

- **Discourse "**Discourse" refers to a reflective interchange between two or more parties. One of the parties might be a computer program that allows the student to investigate the properties of a class of functions as the values of the parameters are changed. On the other hand, all of the parties might be students engaged in cooperative verbal interchange that probes various potential solutions to challenging problems. In either case, the student is striving for sense-making, a notion that is fundamental to understanding mathematical concepts, solving problems, and doing mathematical arguments (proofs). Discourse is fundamental to meaningful learning. A teacher's explanations have limitations. When knowledge is complex, there is too much to tell. You can't tell it all. If you could tell it all, nobody would hang around to listen. Indeed, there is growing research on the role of culture, community and inquiry in learning (Allen and Johnston-Wilder, 2004; Bruner, 1996; Cobb, 1995; Cobb, McClain and Whiteneck, 1997; Cobb, Wood and Yachel, 1990; Cole, 1985; Coles and Brown, 1999; Dubinsky, 1991; Pirie and Schwarzenberger, 1988; Watson, 2002; Watson and Mason, 1998, 2002; Waywood, 1992, 1994; Wheatley, 1992). Such community and inquiry can be supported through the use of a variety of educational technologies (Balestri, Ehrmann and Ferguson, 1992; Baron and Hynes, 1996; Day, 1993; Ferguson, 1992, 1993b; Giamati, 1995; Goldenberg, Lewis and O'Keefe, 1992; Hector, 1992; Smith, Ferguson and Gupta, 2003).

The syllabus for IMDM contains the following note: "Cooperative problem solving will be a major part of this course. Students are expected to attend class and participate in group activities." Fifteen percent of the grade is derived from the full-class and group discussions. Since a lot of mathematics is done in the course, some students think that it is strange to have discussion as such a key part of what they define, at least initially, as a "math course." Some students seem a bit uncomfortable, again initially, by the abrupt shifts that we often make from formal mathematics to contested issues (e.g., local environmental issues) where math is just one part of the picture.

- **Open-ended Problems** The author is using "open-ended" here to refer to two types of problems. One type of problem has specifications that are sufficiently general to give rise to many (possibly infinitely many) valid solutions. A second type of open-ended problem is a problem that is yet unsolved. Let me say that when students are given flexible control over the vocabulary and ideas of a discipline, they will begin to ask hard questions. Some of these questions are not easily resolved, and some may even baffle the teacher. Some problems may be solved but in such a way that only a glimpse of the method used can be conveyed to students in a freshman course. Some problems may not be solved at all. Such use of open-ended problems, with special attention to individual students, supports the constructivist approach (Anderson, 1987; von Glasersfeld, 1989, 1992; Resnick, 1983, 1987; Schauble, 1990; Wiggins and McTighe, 2004, 2005).

I give students a number of different articles, with embedded models, and ask that they comprehend and critique the articles. Some of the articles strike some students as being within their own areas of expertise, and they challenge me on having "crossed over' into a domain where they are the greater experts. Based only on having looked at the title page of one of the articles, a student came up to me with anger in her voice. She asked, "What do you know about the author of this paper? Who is he? Is he qualified to write on this topic?" I must confess that I was unprepared for what I perceived to be a premature attack on the author. Anyway, I asked the student to please read the article and let me know what she thinks. With what I detected as a bit of reluctance, she said, "Okay." At the next class meeting, the student stayed after class to talk to me. She said, in regard to the author, "You know, he knows a lot!" By the way, the paper that the student was referring to is "Hunger, Science and Public Policy" (Hopkins, 2003).

- **Applications** Multiple representations (geometrical, numerical, and algebraic) may be useful in helping students to understand mathematical ideas. In addition to using multiple representations, the author encourages the use of applications to situate mathematics in communities of professional practice. Such context gives students an appreciation for the "messy" environments in which professionals must function.

It is not uncommon for me to find myself in application areas where some students' knowledge exceeds mine by leaps and bounds. One such area is the "Consumer Price Index" (CPI). A political science student in the course asked some deep and hard questions regarding CPI. In many ways, he was rediscovering many of the complex issues that have been raised about how CPI is determined and inventing, at least for himself, some potential solutions.

- **Sustained Effort.** What ever happened to the story line in mathematics? Is there a story to be told? Can each student tell a personal story of the course's major ideas? Sustained work on meaningful projects may be one way to allow students to synthesize knowledge and hence frame their own stories.

Throughout the course, students are encouraged to raise questions about the relationship between core ideas of the course and their personal or disciplinary interests and career goals. The course project on decision- making gives students an excellent opportunity to synthesize an array of ideas and approaches.

- **Challenge** I give students challenging problems. Problems should be sufficiently rich and call on students to utilize a variety of resources: other students, other books, and the computer (or graphing calculator).

Almost without exception, students say that the course is hard; however, they give the course a very positive rating. Indeed, this view is borne out by end-of-semester students' *evaluations of the course. The most frequent comment that students made regarding the course is that the course deals with mathematics in "real" situations. Here is a selection of verbatim comments, taken from end-of-semester course evaluation forms, students made about the relevance of the course:*

"Very informative, actually useful."

"The coding (referring to encryption codes) was very interesting."

"You learn things that are useful in life."

"It is interesting and applicable for life."

"The material is solid and well-rounded, giving us general knowledge."

"I like this course. It clears up confusion about bank policies and the mathematical workings of business."

"Interesting and relevant information."

I was particularly pleased to see that students rated the course very highly in three key areas: "The work is challenging," "I would recommend this course to other students," and "I learned more (from this course than other courses)."

- **Emphasis on Excellence** Students are capable of much more than they can imagine. A key aim of teaching is to elevate expectations and support the student as she/he recalibrates and meets the new challenges.

Since the class was small (about 30 students), I was able to give considerable attention to students. Also, I encouraged students to help each other meet the high standards for the course. Since I did not use any form of curve grading (not even practical with 30 students), students did not perceive the class as a competitive environment. Prior to the start of class, and, of course, *during class, I saw students helping each other with some of the more difficult concepts in the course.*

- **Social Support** Education, especially much of science, technology, engineering and mathematics education, would benefit from greater social support. Such social support would have a tremendous influence on cognitive performance.

If student engagement with each other is an indication of social support, most students became a part of the social learning environment. To my knowledge, only one student remained somewhat isolated from the group. Early in the semester, she told me that groups had not worked well for her in the past. She participated well in the whole-class discussions but remained aloof in groups regardless of the make-up of the group. Anyway, she ended up being one of the top students in the class. Well, I'm not sure whether I need to spend time making sense of this!

- **Mentoring.** Through the problem-solving groups, lots of excellent mentoring occurs. Mentoring is the highest form of teaching. Not all teaching is mentoring, but all mentoring, if it is truly mentoring, is teaching!

It is difficult for me to assess whether effective mentoring took place in this class. Although there was a full range of class levels (freshman through senior) and eighteen different majors represented, it is not clear that the class/course environment made full use of that diversity. The following majors were represented in the group of thirty students: multidisciplinary studies, psychology, history, technological system management, applied mathematics and statistics, Spanish, cinema and cultural studies, political science, theatre arts, anthropology, electrical engineering, mathematics, English, business management, German *languages and literature, mechanical engineering, general/undeclared, and engineering science. Somehow, I think that I could have made better use of this diversity in majors.*

- **Success Behaviors** What does it take to be successful as a mathematics student? Small-group settings provide an excellent opportunity to explore with students what it means and takes to be an academically strong student in quantitative disciplines. Of course, many of the effective behaviors are important outside of quantitative disciplines. We want students to develop this "reflective practice," a kind of self-monitoring of their behaviors in relationship to their goals and objectives.

Critical interventions that occur early in students' undergraduate years can have profound implications for their success in later years. With this in mind, I believe that all teachers of "first-year" students have a special responsibility to help these students develop general abilities that they can use across subjects and throughout their formal education and beyond. In IMDM, I try to help students develop behaviors (responsibility, focus, collaboration, academic integrity, etc.) that can have implications for their professional lives.

- **Student Leadership** A major goal of any educational venture should be to empower students to take control of their academic, social, and personal lives. Both within and outside of class, it is important to support the development of student leadership. Such student leaders can become partners with us in breaking new ground in student services and raising the bar of excellence.

I did not address explicitly the student leadership issue in the course, although, as I noted, some leadership did emerge as students took over the "teaching" role by explaining things they knew from study in their majors and from their work experiences.

- **Learning Through Projects** Projects encourage students to make sense of core knowledge, extend their knowledge and synthesize knowledge into a personally meaningful whole. In the context of IMDM, students must analyze a decision problem in which mathematical and statistical reasoning play a significant role.

Students explored a wide array of decision problems ─ ranging from "How to Control the Deer Population on Long Island" to the "best way to negotiate with the Long Island Railroad (LIRR) for ticket discounts for Stony Brook University students." The "deer" problem was studied by a student who works for New York State parks. The LIRR problem was done by a commuter student who is majoring in Technological Systems Management.

- **Enjoyment** Learning should be fun. In the context of the community of learners, some of that fun is derived from being a part of a supportive learning environment. Another part, a new sensation for many students, is derived from the sheer excitement of understanding and doing mathematics.

I had numerous discussions with students in the course. The following items were mentioned by many students as things that they particularly liked about the course: working on "real" problems, seeing connections between the course and other courses that they have taken/are taking, interacting with other students, realizing that they could do mathematics, and the challenge of trying to succeed on all parts of the course. A couple of students from "quantitative" disciplines felt that the class was not technical enough. By the way, some of the "quantitative" students found it difficult to engage with mathematics when it is embedded in a complex social domain. For some of these students, it was a hard reality to find that mathematics as they had come to know it was just one part in addressing a complex social issue.

I am pleased and encouraged by students' accomplishments in the course. Although I will need to plan for external evaluation of the course, my prior experience with earlier versions of the course suggests that the new direction that I have taken is extremely promising. First, in the reformed course, students' skills on "routine problems" are better than what I have seen before. Secondly, students are showing much greater flexibility in the application of their knowledge to new domains. Thirdly, nearly every student is succeeding in the course. Finally, students are asking me whether there are related courses that they can take in the coming semester. I am anxious to work towards an authentic assessment of the course that would guide its future development.

V. Enduring Influences: Engaging with Mathematics from Positions of Strength

Students who are very capable in other subject areas often fail to engage with mathematics. Often these students see mathematics as a textbook subject that limits their connection to the subject to struggling through notation-ridden text and problems that get their attention only for the sake of doing homework and taking tests. Few of these students are intimately and intrinsically engaged in mathematics or statistics for its beauty, elegance or utility. What, then, are ways of helping students to make sense of mathematical concepts and find connections between those concepts and their core interests and expertise?

Over the last 45 years or so, cognitive scientists have developed a "science of learning" that has tremendous implications for the learning and teaching of mathematics, science, and virtually every other subject. As Etkina and Mestre observe,

> "…the term *learning* within cognitive science is synonymous with understanding, and the study of learning with understanding in cognitive science is often approached from a

multidisciplinary perspective. Current views of learning include the idea that individuals construct knowledge (CPSE, 2003). Learners not only construct knowledge but the knowledge they already possess affects their ability to learn new knowledge. If new knowledge that we are trying to construct conflicts with previously constructed knowledge, the new knowledge will not make sense to us and may be constructed in a way that is not useful for flexible application (Anderson, 1987; Resnick, 1983, 1987; Schauble, 1990; von Glasersfeld, 1989, 1992)." (Etkina and Mestre, 2004).

The principles and implications of the science of learning have now been consolidated in several sources (Bransford, et al., 1999; Bransford and Donovan, 2005) and applied to many domains, including engineering education (Adams, Turns, and Atman, 2003; Atman and Bursic, 1998; Atman, Chimka, Bursic and Nachtmann, 1999; Atman and Nair, 1996; Besterfield-Sacre, Atman, and Shuman, 1998; Center for the Advancement of Scholarship on Engineering Education, 2004; Mullin, Atman, and Shuman, 1999; Turns, Atman, and Adams, 2000). Etkina and Mestre (2004) report on several factors from the science of learning that have major implications for learning. The factors, as presented by Etkina and Mestre, are highlighted in the bullet items below and related to IMDM through remarks in italics.

- Constructivism and the Role of Prior Knowledge in Learning

  Early in the semester, students learn that they cannot do well in the course by resorting to pattern matching (classifying problems into a few categories and fitting every problem into one of the categories) or applying a few standard techniques. These approaches, which may have worked well for students in high school or even in some of their college courses, won't work here. When students realize that they must think through problems, they are inclined to discuss their thinking with other students. It is in such discussions that *misunderstandings and multiple approaches to problems are revealed. One student commented early on in the course that "the book doesn't help us solve a lot of the problems." I think that what he was saying is that the book does not give a "recipe" that can be used in approaching many of the tasks in this course.*

- The Nature of Expertise: Organization and Application of Knowledge

Experts' knowledge is hierarchically organized with major principles/concepts at the top of the hierarchy. Experts give attention to deep structural characteristics of problems rather than surface features.

In mathematics texts and in instructors' presentations on mathematics topics, there is generally a high degree of organization of the knowledge. The problem is that students often cannot see the underlying structure, and, furthermore, they do not take ownership of such externally imposed organization. In IMDM, I use modeling as a central theme, and I try to help students organize their knowledge by focusing on some key issues regarding modeling. In addition, I try to help students frame their own stories so that they can construct personally relevant connections among the myriad of concepts and applied topics in the course. I think that I need to do more to help students frame their own stories about the course.

- Transfer of Learning

In their SENCER Backgrounder, Etkina and Mestre have this to say about the transfer of knowledge:

> "…research suggests that transfer is enhanced when the learner abstracts the deep principles underlying the knowledge being learned and that abstraction is facilitated by opportunities to experience concepts and principles in multiple contexts. People's prior

knowledge and experience in a domain affects their subsequent transfer, although sometimes the effect is initially negative because previously learned concepts and routines must be changed to deal with new settings (e.g., Barnett & Ceci, 2002; Bransford, et al., 1999; Bransford and Schwartz, 1999; Hartnett and Gelman, 1998; Singley and Anderson, 1989)." (Etkina and Mestre, 2004)   [For a current and expanded view, see also Mestre, 2005.]

A major goal of the course is that students use the understandings developed in the course in a myriad of other contexts and roles (other courses, career, and as citizens in a democracy). Although I can see that students do make connections to what may appear to be very different areas within the course, it would be useful to have an assessment done of the "enduring influences," including transfer and attitudinal issues as well as propensity of students to engage with mathematics and statistics in future courses and more informally.

- Metacognition: Self-Reflecting about Learning

There needs to be more emphasis on getting students, at every level and in every subject, to reflect on their learning processes. I'm reminded of a comment made to me a few years ago by a high school student in response to my question regarding how she was doing in her algebra class. The student responded, "I'm getting an 'A' in the class, but I don't understand a thing." However unfortunate from a learning of algebra perspective, the student's response is interesting. Clearly, she has made a distinction in her own mind between "getting a good grade" and reaching a comfort/understanding level with the material. My own sense was that this student was likely to make significant learning gains in the future. In IMDM, I try to help students develop the view that things, including mathematics, are there to be made sense of. I encourage students, through writing and *discussion, to frame their personal views of knowledge and to see how that knowledge "makes sense" of things.*

- Assessment in the Service of Learning

Assessment should be an integral part of the learning process and not simply a summative measure against a standard. Such assessment should be consistent with constructivist views of learning ─ encouraging and rewarding students' reflection on knowledge, generation of questions, and development of their own plans. Recent research has contributed to an understanding of such assessment (National Research Council, 2001).

In IMDM, I find assessment more challenging than I have encountered in more "standard courses." In this course, issues of interpretation/comprehension of models, applying knowledge in new contexts, and building models are key. In whole-class discussions, group work, written assignments, tests (midterm and final examinations) and the course project, I reward students for thinking and the articulation of their thinking. In an early class, a student asked, "Is partial credit given in this class?" I followed up by commenting that in the context of modeling, most/all thinking is partial/tentative. Yes, there is partial credit just in the same sense that one would expect such credit for a critical analysis in history, political science, sociology, and other areas. I followed up that many "arguments" include data and underlying math issues for which there are indeed right and wrong answers. When a total argument gives wrong answers to such components, it weakens the overall argument ─ in the same way that getting the "facts" wrong on a newspaper article on an environmental issue jeopardizes the credibility of the entire article.

In the context of IMDM, I seek to engage students in ways that the experience will have enduring influence. First, a constructivist approach to learning is used throughout. This results in students' engaging in sense-making on the fundamental concepts of the course. Secondly, the course situates fundamental concepts in broader and multiple contexts that encourage students to see how mathematical and statistical approaches inform, misinform, or distract from debate on current issues. Thirdly, students develop their own projects so as to work through fundamental content in personally meaningful contexts.

VI. Design Challenges

There are many ways of "knowing." Mathematics and statistics, art, music, poetry, philosophy, social and natural sciences, and other human endeavors may reflect different purposes and different ways of knowing ourselves and the world. Each of these "ways of knowing" has its assumptions, models and limitations. Learning can be greatly enhanced when we explore common threads that run across many domains and when we contrast methods that are dominant in one domain with prevailing approaches in other domains. Indeed, situating mathematics and statistics, as well as other areas, in compelling contexts brings us head on to disciplinary assumptions, methods and "boundaries."

What does it mean to situate mathematics and statistics in compelling contexts? Mathematicians speak of beauty and elegance (Dunham, 1990; Dunham, 1994; Hardy, 1967; Lang, 1985; Weaver, 2003). It is not enough to say that we want students to appreciate the beauty and elegance of mathematics. The grand challenge is to find ways to have that beauty and elegance speak to students. As in any art form, considerations of individual or group expressions and the extent to which those expressions connect with (or speak to) others may be paramount. Perhaps, situating pure mathematics in the broader context of logic and argumentation may provide the compelling context for many students. In particular, exploring how mathematical system building and mathematical arguments (especially proofs) differ from other linguistic structures and informal arguments may be motivating for many students. Much more attention needs to be given to the design of activities that encourage students to see the beauty of mathematics.

In applied mathematics and statistics, the concept of "model" is extremely powerful. In this context (and perhaps in others as well), a model is considered to be a "tentative" conceptualization of reality that is useful in helping us to understand and/or predict phenomena. Tentative could mean a few days or a few hundred years (or longer). In IMDM, the author uses the concept of modeling as the unifying theme to which we can establish bi-directional crosswalks from the core mathematics concepts to many "applied domains" (history, political science, business, etc.). In particular, many connections are made to the role of modeling in decision making (Baron, 2000; Copp, 1989; Day, Reibstein and Gunther, 1997; Hoch, Kunreuther, and Gunther, 2001; Kahneman, Slovic, and Tversky, 1982; McKenna, 1991; Plous, 1993; Nutt, 2002; Smith, 1990). IMDM is an interdisciplinary course that has mathematical and statistical reasoning as integral and vital dimensions of the course. The mathematics and statistics dimensions are sufficient to have the course included in the list of approved courses that satisfy Stony Brook University's general education requirement on mathematical and statistical reasoning.

We need to explore multiple ways of situating mathematical and statistical reasoning within compelling contexts. The SENCER approach teaches to basic science and mathematics through complex, capacious and unsolved public issues. In IMDM, a significant portion of the courses focuses on exploring mathematical and statistical concepts within more complex societal issues. There is a growing body of research in the related area of "science, technology and society" (STS) that supports such contextual approaches (Hurd, 1986; Kellerman and Liu, 1996; McCormack and Yager, 1989; Penick, 1996; Wilson and Livingston, 1996; Yager, 1996; Yager, 2004; Yager, 2005; Yager and Tamir, 1995; Yager and Weld, 1999). In 1990, the Board of Directors of the National Science Teachers Association defined nine essential features of the STS approach. Yager (2005) lists these features and uses them in his study of the impact of STS on student achievement. In reflecting on the decision-making project that I require in IMDM, I can see that the project addresses nearly all of the key issues outlined by the Board of Directors of NSTA. Before illustrating how the project assignment addresses the nine features, I would like to say a bit about what I require of students for the course project.

Each student is required to do a course project that demands that students formulate and "solve" a decision problem. In particular, students must respond to the following questions/issues in regard to a decision problem involving personal, campus or community organization, or work issues:

(a) What is the decision problem that you are addressing, and why is it important?

(b) Give appropriate background on the project that will allow the reader to understand the context for the problem. Please indicate who the decision makers and clients are.

(c) Please state the alternatives that you are considering, and give a rationale for your specific set of alternatives.

(d) What is your plan for addressing the decision problem? (That is, what approach will you use to determine which alternative is "best?")

(e) What is the role of data (specific data, representation of data, summary of data) and quantitative modeling in your analysis of the decision problem?

(f) Write a report that includes (i) your responses to (a) – (e), (ii) the outcome of the implementation of your plan, and (iii) a detailed discussion of the power and limitation of your decision study as an aid to the decision makers.

I will now illustrate (in italics) how the IMDM project on decision-making addresses each of nine STS features as defined by the Board of Directors of NSTA and outlined by Yager (2005).

- Student identification of problems with local interest and impact.

Each student selects a decision problem connected to personal issues, campus or community organizations, or work. I encourage students to consider problems whose resolution would benefit themselves and others.

- The use of local resources (both human and material) to locate information that can be used in problem resolution.

Students used a wide range of resources in addressing components of their decision problem. Such resources included campus and community "experts" and reports.

- The active involvement of students in seeking information that can be applied to solve real-life problems.

A student who worked for the New York State parks acquired an extensive amount of material on strategies that Long island has been using or plans to use to control wildlife populations. This particular student was especially interested in the role of modeling in the control of wildlife populations. He is now involved in some of New York State's planning in this area.

- The extension of learning beyond the class period, the classroom, and the school.

Student projects took them to diverse areas of the Stony Brook University campus, the Long Island area, and New York City. In an effort to gather data regarding peak periods in which Stony Brook commuter students traveled on Long Island Railroad trains, one student had passengers complete short surveys while they were traveling to work, school, etc. The student planned to use such data as part of an argument to Long Island Railroad as to why it should offer ticket discounts to Stony Brook commuter students.

- A view that science content is more than concepts that exist for students to master for tests.

Students told me that in the context of doing their projects a lot of course concepts came to life: quality of data, generalizability of results, inference, modeling, etc. Generally, students also learned some things about the politics of decision making; that is, when students are challenged to engage with civic issues, they may discover that they encounter some opposition from those who may have different values, interests, knowledge bases, and intentions. From reviewing student projects, I am encouraged that such intimate engagement with core course concepts will have an enduring influence on a number of the students.

- An emphasis upon process skills that students can use in their own problem resolution.

In the context of their projects, students could see that their abilities to plan, gather information, critique information, and articulate their findings are critical.

- An emphasis upon career awareness—especially careers related to science and technology.

I do not have sufficient information to give an informed view as to whether the course project or other aspects of IMDM had an impact on students' understanding of career choices.

- Identification of ways that science and technology are likely to impact the future.

I tried to help students learn to "see" connections between course content and current and emerging societal issues. When I started to discuss Radio Frequency Identification (RFID), a student piped up immediately and proudly to fill us in on Wal-Mart's move towards RFID. Although I knew a bit about this issue, she impressed me and the rest of the class (I think!) with her knowledge of Wal-Mart's business operations. After class, I asked her how she knew so much about Wal-Mart. She said that she had investigated Wal-Mart's business practices in connection with a project management class that she had taken.

- Student autonomy in the learning process as individual issues are identified and approached.

I discovered that there are several nice things about having students do projects. In addition to the items previously mentioned, two other motivations for projects come to mind. First, projects put students in the role of "experts." Indeed, that was the case with students' decision problems. I learned a lot about campus issues, local problems and, of course, students' perspectives from engaging with students on their projects. Secondly, projects often provide a vehicle for students to engage with professionals (practitioners) in a variety of fields. Such engagement can help to bridge the unfortunate gap between "academic knowledge" and the knowledge that evolves in "communities of professional practice."

These features, conceptualized for science and technology, demonstrate a broader context that could have implications for the learning of mathematics.

Some mathematicians have sought to situate mathematics within contexts that allow students and faculty to better understand mathematical thought and see the beauty of mathematics (Dunham, 1990; Dunham, 1994; Hardy, 1967; Lang, 1985; Pólya, 1954; Pólya, 1961, 1965/1981; Weaver, 2003). The non-routine problem solving context is still another approach/context (Pólya, 1945; Schoenfeld, 1985, 1987, 1989b, 1989c, 1992). Within each of these "compelling contexts" (SENCER approach, mathematics as art, problem solving) and many other contexts, there are multiple models of implementation. The learning and teaching of mathematics could be greatly enhanced through articulation of multiple contexts and multiple models for engaging students within a "compelling context." Through educational research, we will be able to assess the effectiveness of the various contexts and models for achieving specific learning goals.

Mathematics and statistics have many faces. Making conjectures, formulating definitions, proving theorems, building systems, solving problems in "abstract" setting, and developing and using models are some of those faces. It is extremely important to build compelling contexts that allow students of diverse backgrounds, interests and achievement levels to build bi-directional crosswalks from core mathematical ideas and a plethora of seemingly different worlds.

VII. Conclusion

Mathematicians, scientists and engineers argue that there are so many compelling reasons why mathematics ought to be an integral part of students' education at every level. We get poetic about beauty, elegance and/or utility. Indeed, we are joined by professionals outside of "quantitative disciplines" and by concerned citizens throughout our nation in advocating for students' enhanced participation in and performance in mathematics. There are constant reminders that U.S. students will enter a competitive global workforce. In addition, there is concern that inadequate preparation in mathematics threatens to undermine students' effective participation in a democracy. Emphasis on standards and testing not withstanding, many capable students remain disengaged from mathematics. That is, in spite of so many "compelling reasons" why students ought to be engaged with mathematics, these compelling reasons have not yet become compelling from the perspective of the students.

This paper calls for increased emphasis on a vision of mathematical and statistical reasoning that situates these subjects in compelling contexts (compelling from the students' viewpoints) and thereby allows for the development of core mathematical concepts that can be interconnected to a variety of interests and purposes. That vision suggests a modest core of mathematical concepts and methods, developed through a constructivist approach. Students will build on that core to both extend their understanding to new mathematical ideas/approaches and enhance their understanding of complex (dirty) domains in which mathematics contributes to their evolving knowledge. This approach recognizes that, both within the core and beyond the core, students' knowledge will evolve in different ways. Such student-initiated learning paths will demand creative and flexible assessment methods that not only gauge progress on benchmarks but reveal insights about learners' unique experiences on personally meaningful projects.

The vision of mathematical and statistical reasoning presented in this paper has implications far beyond that of teaching students who have interests in "non-quantitative" disciplines. All students, at every level (elementary school through graduate school), might greatly benefit from an environment that supports the learning of mathematics in several ways: uses a constructivist approach to engage students in a core set of mathematical ideas, methods and systems; provides students with a wealth of resources and encourages students to seek other resources to fill in gaps in their knowledge and expand learning opportunities; creates a community of learners where students and faculty offer cognitive and affective support to each other; creates multiple, bi-directional intellectual crosswalks that connect core content to many domains; and encourages students to build on core knowledge by defining their own pathways to new domains. Such a dynamic approach would help students to see the big picture and their evolving role in it.

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