I regularly assign homework problems from the textbook. Students read out of the book *Collapse: How Societies Choose to Fail or Succeed* by Jared Diamond, and write short response papers in which they describe the ways that they see the material in our math course applying to the social issues being discussed in the chapter. There is a more focused assignment on over-population and the Rwandan genocide (See Appendix for Rwanda Assignment). There is a final project in which student teams learn about a topic of interest that involves differential equations, give a short oral presentation on their project and write a 10 – 15 page report on their findings. (See Appendix for description of final project and list of potential project topics.)

We have a special three hour class meeting one evening in which we learn about over harvesting of resources by playing the simulation game Fishing Banks, Ltd created by Dennis Meadows. (See Appendix for Fishing Simulation Game).

Class Schedule
Below is the course "play-by-play" in which I briefly describe the topic for each class and also have links to the handouts for group work and computer work that we used in class that day. Also below is an example of a group modeling project.

Wk 1: Mon Jan 23
HW: Calculus Review (40.496 Kb)
Student Info Form (45 Kb)
Pre-course Assessment (48.143 Kb)
Syllabus (47 Kb)
Overview of course.
For Wed: take pre-assessment (do this before doing the homework; please do not review for the pre-assessment); read the syllabus, read "A Note to the Student" p. xiii in textbook, do problems on homework assignment.

Wk 1: Wed Jan 24
Worksheet(Exponential Fns, DE, Populaton growth) (25.5 Kb)
Supplementary HW problems (27.922 Kb)
US Population Data Table (35.288 Kb)
Reviewed solutions to DE related to exponential functions (on worksheet); initial value problems; general solution. Given data, how to use model arising from differential equation, to predict future behavior. Example using US population data (see table). How good is the model? Absolute and relative error. Doubling time for exponential function. Model for radioactive decay.
HW due on Mon Jan 29th: Sect 1.1 #1, 6 (challenge problem), 10 (see bottom of page 16), 11, 12ab.
Also, examine the table of US population and discuss whether you think the model used to predict the population is a good model.
Finally, there are a few integral problems to do on the Supplementary HW sheet.

Wk 2: Jan 29
Modeling Worksheet (20.5 Kb)
Mini Quiz Jan 29 (31.704 Kb)
Graphing exponentials (review sheet) (66 Kb)
Rwanda and population growth (1.313 Mb)
Rwanda population data table (35 Kb)
When is a model "good"? Sect 1.2: separation of variables to solve DE in systematic way. Practice with
translating sentence into a math equation (worksheet).

HW for Wed:
a. Sect 1.2 # 6, 7, 9, 16, 23, 25,
b. Try to make the model (ie the differential equation) for the cooling problem. It is on the worksheet handout above.
c. Do mini quiz and hand this in separately to Prof. Donnay.
d. If you did not show Mastery for graphing exponentials on the pre-assessment, do the work sheet on Exponentials. (Even if you showed mastery, if you would like a review of exponentials, you can do the worksheet).
HW for next Mon (more will be assigned on Wed):
Sect 1.2 # 8, 10 (review inverse trig!), 11, 13, 27, 30, 35
- Make an estimate of the total number of people who could possibly live in the United States. Explain clearly the assumptions you make in producing your answer and show your work clearly.

Do Rwanda reading and answer:
1. What does the book claim is the relative growth rate of the Rwanda population (k value)?

2. What would be the doubling time with this relative growth rate?

3. Looking at the population data table, calculate the relative growth rate (k) of population using the data from 1980 and 1985.

4. With this k value, predict the population in 1995. Does your prediction agree with the value given in the table? Discuss reasons for the difference.

5. Write a paragraph describing some of the ways that over population in Rwanda caused problems.

Wk 2: Wed Jan 31
Calculating slope fields and finding patterns (22 Kb)
Additional HW for Mon Feb 5th (33.324 Kb)
Calculating slope fields and drawing curves to match slope field. (5.755 Mb)
Modeling - translating sentence into DE: lessons learned;
- give names to variables and write down what the variables represent
- give units

- "rate" means a derivative
- "proportional" means there will be a constant of proportionality.
- draw a diagram/picture. Represent the information from the problem in the diagram.

Sect 1.3: Graphical solutions of DE: initial value corresponds to "clicking point" on computer. Many different solutions to DE depending on IC. Equilibrium solution = constant function. Attracting (stable) equilibrium solution. System evolves towards equilibrium as time increases.

Graphing solutions to match slope field. Calculating and graphing slope fields. y = f(x,y). Autonomous DE: y' = f(y) - the slopes are constant on each horizontal line (since y = constant there). Separable DE: y' f(t, y) = g(t) * h(y). Can solve such DEs by separation of variables technique.

HW for Monday Feb 5th: In addition to what is listed in last class, also do those that are on HW attachment.
HW for Wed Feb 7th: Sect 1.3 #15.

Wk 3: Monday Feb 5
Collapse - Easter Island (2.788 Mb)
Instructions for Euler's method excel spreadsheet (36.877 Kb)
Excel spreadsheet for Euler's method. The user enters all the data (34.5 Kb)
More advanced spreadsheet for Euler's method: makes automatic some of the steps (35 Kb)
Euler's method group worksheet (33 Kb)
QuizFeb5 due next class period (32.111 Kb)
Review of HW. Euler's Method (Sect 1.4) including group worksheet and Excel spreadsheet.

For Wed Feb 14th (next week), read the chapter on Easter Island and its collapse. Write a half page "reaction" to the reading - what caught your interest?

Class on Wed Feb 7th has been rescheduled for Friday Feb 9th 2:30 - 4pm; same room. Hw that was due on Wed Feb 7th is now due on Friday Feb 9th.

HW for Friday Feb 9th: Sect 1.3 #15. Sect. 1.4 #3. Do this problem both by "hand" via a table as we did in class and by using Excel. Print out a copy of your Excel spreadsheet and XY Scatter Plot (or even better - using the Line plot - Christina can show you how) and hand in with homework. Finally, do again via your Excel spreadsheet but now with a step size of Delta t = .01. Again hand in both your spreadsheet and plot. In all cases, you should summarize your work by stating clearly what your final answer is.
Redo: please do Sect 1.3 # 14 (it was due in the last hw) and hand in on Friday.

Quiz due on Friday too.

Week 4, Friday Feb 9
Class notes (69.146 Kb)

Homework due Mon Feb 12 (33.118 Kb)
Line Plot for Excel: graphing y as a fn of t (19 Kb)
Practice with translating word problem (Mixing) into a differential equation (see homework assignment sheet). Logistic equation (p. 9-11).
Graphing solutions and phase lines. Use the HPGSolver program to examine solutions to the differential equation dy/dt=2y(1-y/10). Translate the slope field diagram into a phase line picture (Sect. 1.6).

Attached are notes from today's class - unfortunately the writing is faint but they should give you an idea what we covered.

Also attached are instructions on how to plot (t, y) from an Excel spreadsheet using the line plot feature.

Wk 4: Mon Feb 12
Group worksheet to draw phase line; linking phase line to slope field and solution curves (26.5 Kb)
HW for Wed Feb 14 (30.562 Kb)
Quiz due Wed Feb 14 (29.555 Kb)
Discussion of separation of variables: be sure to do u, du substitution for ln integration. Review of quiz question. Then same idea behind population model with immigration. Compartment model for mixing. Link to SRI model of diseases spread (mathematical epidemiology). Get system of 3 interdependent autonomous differential equations. Group worksheet about equilibrium, slope fields, phase line.
Homework due Wed (file above).

Quiz due Wed

Wk 4: Wed Feb 14
Table to use in recording instances of math in the chapter on Easter Island from the book Collapse. (29.5 Kb)
Worksheet to study population growth with harvesting (29.456 Kb)
HW due on Monday Feb 19th (25.722 Kb)
Discussion of Easter Island reading. Worksheet to record examples of math/quantiative methods used in the chapter. Due Monday Feb 26.

Population growth with harvesting: worksheet to be done for Mon Feb 19.
Handouts related to fish simulation excercise on Sunday Feb 18 at 3:45- 6;15 in rm. 354 (computer lab).

Existence and Uniqueness of solutions of ODE (Section 1.5)

Copies of the worksheet, handouts are on shelf outside Prof Donnay's office. The Fish Simulation handout is not on blackboard; the other handouts are there.

Homework due this Monday is attached.

Wk 5: Monday Feb 19
Fishing game results 1 (17 Kb)
Fish Game results 2 (36 Kb)
Discussion of Fishing Game; looking at data about fish game (attached) to see that fish population crashes before the fisherman are aware of it. By the time you notice it, it is too late!
Groups discussed logistic growth with harvesting for their c values.

Sect 1.5 (Existence and Uniqueness). We discussed main ideas. Look at statement of theorems in book for 'formal' statements. Do you see the relation between our discussion in class and the formal statements?

HW: hand in on Wed the hw that was due on Monday. Plus:
- write 1/2 - 1 page reflection on the Fishing Game
- draw on one graph the f(p) curves as the value of c changes. Try to do all 8 graphs on one picture (if this turns out to be really hard, do at least your two c values).
- Fun challenge (be a mathematician): think of math questions you could ask about equilibrium and solutions of autonomous DE. Next Monday we will see what questions you have come up with.

Wk 5: Wed Feb 21
Harvesting-Bifurcation Diagram Gp worksheet (34.5 Kb)
HW due Monday Feb 26 (23.244 Kb)
Assignment to study effect of various harvesting levels for fish population (25.722 Kb)
Sect. 1.5 Finish discussion of existence and uniqueness by looking at formal statements of theorems.

Sect 1.7: Systems that depend on a parameter (C). How the phase line picture varies as the parameter (C) varies. Putting all this information together into a bifurcation diagram. (Group worksheet).

The midterm (closed book takehome) will be given out next Wed Feb 28th and will be due back at the start of class on the following Monday.

Wk 6: Wed Feb 28
Linearity and Linear DE (51.773 Kb)
On Monday Feb 26, worked on HW.
On Wed Feb 28, reviewed HW and started linearity (see attached worksheet)

Wk 7: Monday March 5
Hw due Wed March 7 (27.5 Kb)
Sheet for feedback on class (20.5 Kb)
Continued discussion of linearity. Looked at linear (and non-linear) differential equations. Sect 1.8. General solution of a non-homogeneous linear equation is a sum of the general solution of homogeneous solution + any one particular solution of the

homogeneous equation.
Did examples from worksheet - given out last class.

Wk 7: Wed March 7
Hw due Wed March 21 (27.944 Kb)
Computer exploration of systems of differential equations: worksheet (26.952 Kb)
Student observations about systems of differential equaitons (14.632 Kb)
Group discussion on quantiative issues in Easter Island reading.

Solutions of linear non-homogeneous equations. Proof that general solution is a sum of the general solution of the homogeneous equation plus one particular solution of non-homogeneous equation.

Idea of a Differential Operator acting on a function space :)
A linear differential equation translates into this operator satisfying the linearity properties (1) and (2).

Introduction to systems of 2 differential equations. Computer exploration (worksheet) , observations, discussions and connections with slope fields.

Midterm Test redos: aim to return them for Wed March 21. May look in notes, text book or talk with Prof Donnay. Please do not ask TA or talk with classmates.

HW For Monday March 20: Review about parametric equations from calculus. Then let x(t) = cos(t) and
y(t) = sin(t). For t in [0, 4 Pi], draw the graph (t, x(t)), the graph (t, y(t)) and the graph (x(t), y(t)) = parametric equations.

Wk 8: Monday March 19
Caution with numerical solutions (16.878 Kb)
Some comments on midterm. (26.5 Kb)
Draw Vector Field (27.939 Kb)
HW: Parametric Equations (x(t) = sin(t), y(t) = cos(t)) (45.5 Kb)
Plot solution curves (21.708 Kb)
Office Hours this week: Tuesday March 20; 3-5pm; Friday March 23, 3-4:30pm.

Test Redo: Extension given - now due on Monday March 26. Comments about rewrites attached.

Review of Numerical solutions with Euler method - accuracy depends on small step size. With too large a step size, numerical solution may not be a good approximation to true solutions. (handout to be posted soon). See problem #6, p. 63 and #13, p. 64.

Discussion of steady state soluiton to non-homogenous differential equation (Sect 1.8, p. 120-121).

Geometry of solutions of systems of differential equations (Sect 2.2). Drawing solutions curves whose tangent vectors (lhs of DE) match the vector field (rhs of DE). Calculating the vector field (rhs) associated with a system.

Parametric equations; relating the graphs of (t, x(t)), (t, y(t)) and (x(t), y(t)). Used the DESketchPad program.

Additional HW for Wed: complete the worksheet involving the parametric equations (x(t) = sin(t), y(t) = cos(t)) and the tangent vectors to this curve.

Wk 8: Wed March 21
HW due Wed March 28 but you should do part of it by Monday March 26 (22.617 Kb)
Sect 2.1: Review of parametric curve (x(t) = cos t, y(t) = sin t) : circle of radius 1: x^2+ y^2=1

Predator-Prey: decoupled system (Sect 2.3), full equations (p. 11-13).
Equilibrium solutions Sect 2.2. Types of equilibrium: attractor, repeller, saddle, center.

Test redo is due Monday March 21.

Wk 9: Monday March 26
Graph paper for plotting vectors (14.559 Kb)
Geometric interpretation of Av. (30.5 Kb)
Determining equilibrium solutions for non-linear systems: solving system of non-linear equations.
What type are the equilibrium? Compare to the basic types given by the linear system (attractor, repellor, saddle, center). The linear systems are the building blocks that will generate the non-linear; so we focus on linear systems for a while.

Spring oscillation: modeled this using a second order DE. Translated a second order DE into a first order system.

End: Calculate v and Av and plot them from worksheet.

Wk 9: Wed March 28
Part of HW: Examine relationship between type of equilibrium for linear system and eigenvalues (30 Kb)
Homework due Wed April 4th (22.837 Kb)
Went over vector multiplication worksheet: plotting v and Av either both at the origin or Av starts at the end of v=(x,y). This later approach is how we create the vector field for the system (dx/dt , dy/dt) = A(x,y).
Eigenvector (gets sent to itself via A) and Eigenvalue = stretch factor. Any multiple of an eigenvector is also an eigenvector.

Eigenvectors of matrix A give rise to "criss-cross" lines in the phase plane picture. Each such line is an eigenline made up of all multiples of an eigenvector. So eigenvector plays a key role in solutions of linear systems.

Next step: what role do eigenvalues play in the solutions (see associated homework worksheet).

Wk 10; Monday April 2
Sections 3.1 and 3.2:

Using matrix notation to study a system: dY/dt = AY.
Checking if a function Y(t) solves the system. Straight line solutions given by

Y(t) = e^(lambda t) v

where v is the eigenvector and lambda is the eigenvalue.

Linearity Principle; linear combination of solutions is a solution. If the initial conditions of these solutions are not multiples of one another; then this combination gives the general solution.

HW Revision: Hand in the hw component on linear systems separately from the rest of your hw.

In Sect 3.2 #2 and 7: do parts (c), (d) ,(e). I am giving you the eigenvalues and eigenvectors . We will learn how to calculate them next class.
#2: eigenvector 1 = (2,1), Eigenvalue 1 = - 5;
eigenvector 2 = (-1,1), Eigenvalue 2 = - 2;

#7: eigenvector 1 = (4, 1), Eigenvalue 1 = 4;
eigenvector 2 = (-1,1), Eigenvalue 2 = - 1;

Midterm 1 Grades: A >= 95% (114); A- >= 90% (108); B+ >= 85% (102);
B >= 80% (96); B- >= 75% (90), C+ >= 70% (84); C >= 60% (72); C- >= 55% (66); D >= 50% (60).

Wk 10: Wed April 4
Hw work due Wed April 11 (23.694 Kb)
Review of straight line solutions (computer has trouble drawing these accurately) and general solution.

Discussion of relation between eigenvalues and types of equilibrium.

Calculation of eigenvalues, eigenvectors.

Proof the Y(t) = exp(lamda t) v is a solution of the system dY/dt = AY
where v is eigenvalue and lambda is eigenvalue.

Wk 11: Wed April 11
Complex Numbers hw sheet (46.363 Kb)
Graph paper (14.559 Kb)
Hw due Wed April 18 (13.611 Kb)
Instructions for browsing thru projects (26 Kb)
Complex eigenvalues lead to spirals (Sect 3.4). Mathematics related to complex eigenvalues and complex solutions; Get two real solutions from Re and Im part of a complex solution.

Wk 11: Monday April 9
Sect 5.3: How to use eigenvalue, eigenvector information to draw the phase portrait.

Sect 5.1: Linearity of matrix transformation; Linearity Principle for solutions of linear systems.

Discussion of projects (see project section above).

Review of basics of DE.

The hw from Sect 3.4, originally due on Wed, will be postponed for a week.

Wk 12: Monday April 16
Graph Linear Phase portraits (30.939 Kb)
Mathematica program to plot approximations (107.67 Kb)
Secct 3.4: Complex eigenvalues. Discuss spiraling; how to determine the direction of motion. Center = Re (eigenvalue)= 0.

Discuss the bending of solution curves for sinks/ sources.
Approximate a function using Taylor Polynomials.

For Wed: - hand in a paper copy of your project description.
- do review worksheet for linear phase portraits.

Wk 12: Wed April 18
HW Due Wed April 25 (35.533 Kb)
Review of Taylor Series.
Peer assessment of linear systems. Discussion of confusing issues.
Introduction to non-linear systems.

Please try to email Prof. Donnay by Friday what project you would like to work on and if you have a partner.

Also, should do the first part of the hw by Monday so you will be better able to follow the next steps in analyzing non-linear systems.

Wk 13: Monday April 23
Sect 5.1: Calculating the linear approximations to a non-linear system near an equilibrium point. Two methods: change of variables and Jacobian matrix (linear approximation).

HW Corrections:

There was a typo on the hw sheet. The system in problem 4 is:

x' = x- y -x^2+y^2 = (x-y)(1-x-y)
y' = 2x+xy = x(2 + y)

In 4a, you should show how you can solve for the equilibrium points analytically (ie using the formulas). You can check that your formulas give the same 4 values we calculated in class.

Wk 13: Wed April 25
Group worksheet for non-linear system analysis (26.5 Kb)
Project Guidelines (32.5 Kb)
Review of non-linear system homework.
Review session for test on Friday 3-5pm. room to be announced.